Unit-4 (Maths 4)

Statistical Techniques-II

Topic :Random Variable

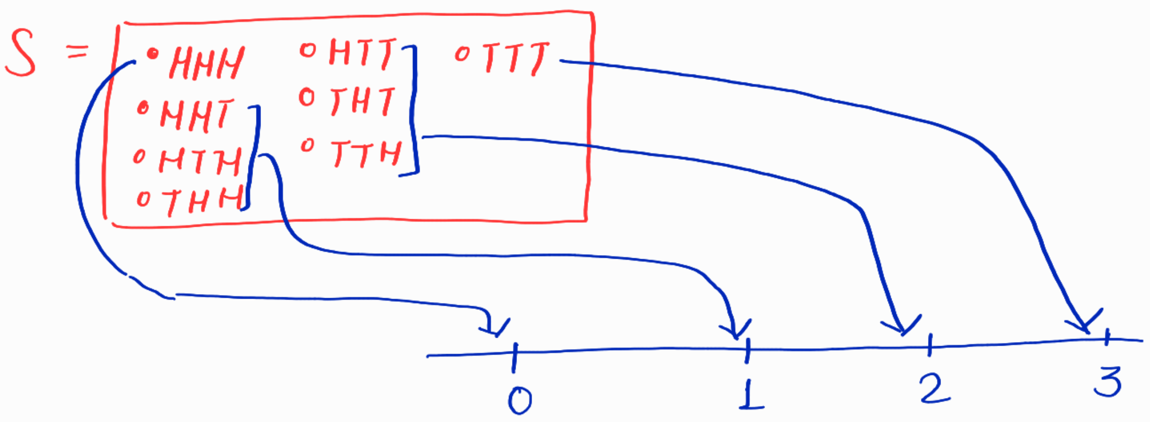
A random variable is real valued function whose domain is sample space of random experiment.

$$X : S \to R$$

[Random Var
Binomial
Poission
Normal
Baye's Th] 20

Eg → Toss a coin 3 times

X = # of Tails



X = assign a numerical value to each outcome of sample space.

$X = \{0, 1, 2, 3\}$

	X = no of Tails $X = x$	Probability $P(X=x)$
HHH	0	$P(0) = P(\text{No tail}) = \dfrac{\text{No of fav case}}{\text{Total case}} = \dfrac{1}{8}$
HHT, HTH, THH	1	$P(X=1) = 3/8$
TTH, HTT, THT	2	$P(X=2) = 3/8$
TTT	3	$P(X=3) = 1/8$

Prob distribution

Types
- Discrete R.V. → Can take countable No of values $X = 0, 1, 2, 3, 4, 5 \ldots$
- Continuous R.V → Can take any value in continuous interval $1 \leq x \leq 2$

Discrete R.V

Probability Mass function (PMF)

let $x_1, x_2, x_3 \ldots\ldots x_n$ be discrete R.V X and $p_1, p_2, p_3 \ldots\ldots p_n$ be Prob corresponding to

$x_1, x_2, x_3 \ldots x_n$ resp

$$\begin{matrix} x_1 & x_2 & \ldots & x_n \\ \downarrow & \downarrow & & \downarrow \\ p_1 & p_2 & \ldots & p_n \end{matrix}$$

$$P(X=x_i) = p(x) = \begin{cases} p(x_i) & i=1,2,3 \ldots \\ 0 & \text{otherwise} \end{cases}$$

Satisfies two properties

$$\begin{cases} \text{(i)} & p(x_i) \geq 0 \;\checkmark \\ \text{(ii)} & \sum p(x_i) = 1 \;\checkmark \end{cases}$$

Continuous R.V $\quad 1 \leq x \leq 2$

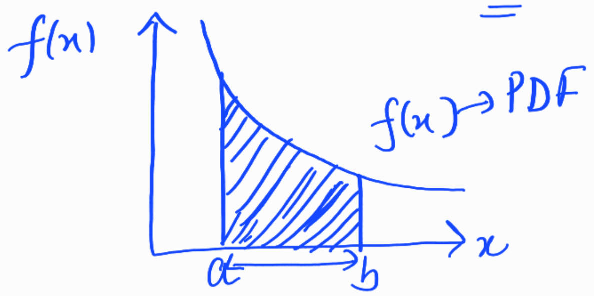
Probability Density function (PDF)

$$P(a \leq X \leq b) = \int_a^b \underbrace{f(x)}_{\text{P.D.F.}}\, dx = \text{Area Under curve of } f(x) \text{ b/w } a \,\&\, b$$

it satisfies two Properties

✓ (i) $f(x) \geq 0$

✓ (ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$



Ex. If fn $f(x)$ is defined by $f(x) = ce^{-x}$, $0 \leq x \leq \infty$, find the value of c, which changes $f(x)$ to a P.D.F.

Sol $\quad f(x) = ce^{-x}, \quad 0 \leq x \leq \infty$

it will satisfy $f(x) \geq 0$

$$\Rightarrow ce^{-x} \geq 0$$

e^{-x} always +ve $0 \leq x \leq \infty$, $\Rightarrow \boxed{c \geq 0}$ ✓

(ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$$\Rightarrow \int_0^{\infty} ce^{-x}\, dx = 1 \Rightarrow c\left[\frac{e^{-x}}{-1}\right]_0^{\infty} = 1$$

$$\Rightarrow c\left[\cancel{\frac{e^{-\infty}}{-1}} + e^{-0}\right] = 1$$

$$\Rightarrow \boxed{c = 1}$$

$\boxed{c = 1} \leftarrow$ Ans

Ex A random var X takes values 1, 2, 3 ------ with p.M.F $\dfrac{\lambda^r}{\lfloor r}$, $r = 1, 2, 3$ ---- Find the value of λ

Sol $\Sigma p_i = 1$ $\qquad$ $PMF = \dfrac{\lambda^r}{\lfloor r} = P(X_r)$

$$\sum_{i=1}^{\infty} p(x_i) = 1$$

$$\Rightarrow p(x_1) + p(x_2) + p(x_3) \text{----} = 1$$

$$\Rightarrow \frac{\lambda}{\lfloor 1} + \frac{\lambda^2}{\lfloor 2} + \frac{\lambda^3}{\lfloor 3} \text{-------} = 1$$

$e^x = 1 + x + \dfrac{x^2}{\lfloor 2} + \dfrac{x^3}{\lfloor 3} + \text{-----}$

$$\Rightarrow \underbrace{1 + \frac{\lambda}{\lfloor 1} + \frac{\lambda^2}{\lfloor 2} + \frac{\lambda^3}{\lfloor 3} \text{-----}} = 1 + 1$$

$$e^{\lambda} = 2$$

$$\boxed{\lambda = \log_e 2}$$

Mean and Variance of Random Var

Let X be R.V. discrete

X :	x_1	x_2	x_3	---	x_n
$P(X)$:	p_1	p_2	p_3	---	p_n

$$\text{Mean} = \mu = \underset{\text{expected Value}}{E(X)} = p_1x_1 + p_2x_2 + p_3x_3 \cdots + p_nx_n$$

$$= \sum_{i=1}^{n} p_i x_i$$

$$\underline{\underline{\text{Mean}}} = \mu = \underline{\underline{E(X)}} = \sum_{i=1}^{n} p_i x_i \quad , \quad \boxed{\sum p_i = 1}$$

Also called <u>first Moment</u> <u>about</u> origin

$$E(X^r) = \sum p_i x_i^r$$

↳ r^{th} Moment about origin

$$\underline{\text{Variance}} = \sigma^2 = \sum p_i (x_i - \mu)^2 = \sum p_i x_i^2 - \mu^2$$

$$= E\{(X-\mu)^2\}$$

$\mu \rightarrow$ Mean

$$\sigma^2 = E\{(X-\mu)^2\} = \sum p_i (x_i - \mu)^2 = \boxed{\sum p_i x_i^2 - \mu^2}$$

$$= E(X^2) - \{E(X)\}^2$$

$$= \underline{\underline{\mu_2}} \text{ (Moment about mean)}$$

$$S.D = \underline{\underline{+\sqrt{Var}}}$$

Unit-4(Maths-4)Lec-2

Statistical Techniques-II

Topic: Binomial Probability Distribution

There are two type of theoretical probability distribution

* Discrete probability distribution → Binomial, Poission, geometric ----

* Continuous probability distribution → Normal, exponential

In Binomial probability distribution, there are n independent trials in an experiment . Let p be probability of success and q be probability of failure in a single trial (p+q=1).
Let X be random variable which denote no. of succes

$$P(r) = \boxed{P(X=r) = {}^{n}C_r (p)^r (q)^{n-r} \quad r=0,1,2\ldots n}$$

$P(X=1) =$ → Binomial Distribution

$p \to$ prob of success
$q \to$ " " failure.

$$P(X=0) = {}^{n}C_0 p^0 q^n$$
$$P(X=1) = {}^{n}C_1 p^1 q^{n-1}$$
$$P(X=2) = {}^{n}C_2 p^2 q^{n-2} \quad \ldots \quad P(X=n) = {}^{n}C_n p^n q^0$$

${}^{n}C_0 q^n, \; {}^{n}C_1 p q^{n-1}, \; \ldots\ldots \; {}^{n}C_n p^n$ are coeff.

of $(q+p)^n$ binomial expansion.

In Binomial Distribution
→ (n), no. of trials is finite.
→ all trials are independent
→ each trial has only two outcomes, success or failure.

Recurrence Relation →

$P(1) = ?$ $P(0)$ known
$P(2)$

$$P(r+1) = \frac{n-r}{\ldots} \frac{p}{q} P(r)$$

$r = 0, 1, 2, \ldots n$

$P(r+1) \quad r+1 \quad q \qquad r = 0, 1, 2 \ldots$

$n \to$ trials

Mean of Binomial Distribution $\to$ $P(r) = {}^{n}C_r\, p^r q^{n-r}$

$$\mu = np \quad , \quad n \to \text{No of trials}, \quad p \to \text{prob of success}$$

Variance of Binomial Distribution $\to$

$$\sigma^2 = npq \quad , \quad q \to \text{prob failure}$$

$$S.D. = \sqrt{Var} = \sqrt{npq}$$

Ex.1 $\to$ Comment on following statement, for Binomial distribution, mean is 6 and Variance 9

Binomial Distri, $\longrightarrow$

$$\mu = np = 6 \quad — (1)$$

$$Var = npq = 9 \quad — (2)$$

$$6q = 9$$

$$q = 9/6 = 3/2 = 1.5 > 1 \quad \text{Not Possible}$$

prob of failure $\qquad$ Prob ≤ 1

Statement False

Ex.2 A die is thrown five times. If getting odd number is success, find Probability of getting at least four success.

$S = \{1, 2, 3, 4, 5, 6\} \qquad n = 5$

Fav Cases $= \{1, 3, 5\}$

$$p(\text{odd No}) = \frac{\text{No of fav cases}}{\text{Total cases}} = \frac{3}{6} = 1/2$$

$$\begin{bmatrix} p(\text{success}) = 1/2 \\ q(\text{failure}) = 1/2 \end{bmatrix}, \quad p+q = 1$$

$$\boxed{P(x=r) = {}^nC_r\, p^r q^{n-r}}$$

$$P(X \geq 4) = P(X=4) + P(X=5)$$

$$= {}^5C_4 \left(\tfrac{1}{2}\right)^4 \left(\tfrac{1}{2}\right)^{5-4} + {}^5C_5 \left(\tfrac{1}{2}\right)^5 \left(\tfrac{1}{2}\right)^{5-5}$$

$$= \left(\tfrac{1}{2}\right)^5 \left[{}^5C_4 + {}^5C_5\right]$$

$$= \left(\tfrac{1}{2}\right)^5 [5+1] = \frac{6}{2^5} = \boxed{3/16}$$

$$\boxed{{}^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r} \qquad {}^5C_4 = {}^5C_1}$$

Ex.3 A binomial Variable X satisfies the relation $\boxed{9P(X=4) = P(X=2)}$ when $n=6$. Find the value of parameter p and $P(X=1)$

Sol

$$\boxed{n=6}$$

$$P(X=r) = {}^nC_r\, p^r q^{n-r}$$

$$P(X=4) = {}^6C_4\, p^4 q^{6-4} = {}^6C_4\, p^4 q^2$$

$$P(X=2) = {}^6C_2\, p^2 q^{6-2} = {}^6C_2\, p^2 q^4$$

$$\boxed{p+q=1} \qquad q = 1-p$$

$$\boxed{{}^6C_4 = {}^6C_2} = \frac{\overset{3}{\cancel{6}} \times 5}{\cancel{2}} = 15$$

$${}^nC_r = {}^nC_{n-r}$$

Given

$$9P(X=4) = P(X=2)$$

$$\Rightarrow 9\,{}^6C_4\, p^{\cancel{4}\,2}\, \cancel{q^2} = {}^6C_2\, \cancel{p^2}\, q^{\cancel{4}\,2}$$

$$\Rightarrow 9\, \cancel{{}^6C_4}\, p^2 = \cancel{{}^6C_2}\, q^2$$

$$\Rightarrow 9p^2 = q^2$$

$$\Rightarrow 9p^2 = (1-p)^2 \Rightarrow 9p^2 - (1-p)^2 = 0$$

$$\Rightarrow 9p^2 - [1 + p^2 - 2p] = 0$$

$$\Rightarrow 9p^2 - 1 - p^2 + 2p = 0 \Rightarrow \boxed{8p^2 + 2p - 1 = 0}$$

$$\Rightarrow (4p-1)(2p+1) = 0$$

$p = 1/4$, $\boxed{p = -1/2}$ X

$\boxed{p = \frac{1}{4}}$ ✓ $\longrightarrow$ $q = 1 - p = 1 - 1/4 = 3/4$

$$P(X=1) = {}^nC_r \, p^r q^{n-r}$$

$$= {}^6C_1 \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^5$$

$$= 6 \times \frac{1}{4} \frac{3^5}{4^5} = \boxed{.3559}$$

Practice Question

① If Probability of hitting target is 10% and 10 shots are fired independently. What is the prob. that the target hits at least once?

[Ans:- .6513]

Unit-4(Maths-4)Lec-3

Statistical Techniques-II

Topic: Binomial Probability Distribution(part-2)

$$\boxed{P(X=r) = {}^nC_r\, p^r q^{n-r}}$$

$r = 0, 1, 2, 3 \ldots n$

p = prob. of success

q = prob. " failure

Ex 1 Fit a Binomial Distribution to the data ← [2012]

x:	0	1	2	3	4
f:	24	41	28	5	2

Sol

x	f	fx
0	24	0
1	41	41
2	28	56
3	5	15
4	2	8
	$\Sigma f = 100$	$\Sigma fx = 115$

Mean $= \mu = np$

$\mu = \frac{\Sigma fx}{\Sigma f} = \frac{\Sigma fx}{N}$

$$\boxed{\mu = \frac{115}{100} = 1.15}$$

$\boxed{n = 4}$

$\mu = 1.15 = 4 \cdot p$

$p = \frac{1.15}{4} = .2875$

$\boxed{p \approx .3}$

$\boxed{q = .7}$

Binomial Distribution

$= N(q+p)^n$

$$\boxed{= 100\,(.7 + .3)^4}$$

Correction

$\Sigma fx = 120$, $\mu = 1.2$

$1.2 = 4p \Rightarrow p = .3$

$q = .7$

Ex.2 In 800 families with 5 children each, how many families would be expected to have

Mgmb

(i) 3 Boys, 2 Girls [AKTU 2017]

(ii) 2 Boys, 3 Girls

(iii) No girl

(iv) At most 2 girl

$N = 800$

Assume prob for boy and girl will be equal.

Sol $\boxed{N = 800}$

$$p(B) = 1/2 = p, \quad p(G) = 1/2 = q$$

(i) $P(3B\ 2G) = P(r=3) = {}^nC_r p^r q^{n-r}$

$$= {}^5C_3 \left(\tfrac{1}{2}\right)^3 \left(\tfrac{1}{2}\right)^2 = {}^5C_3 \left(\tfrac{1}{2}\right)^5$$

$$= \frac{5 \times 4 \times 3}{3 \times 2 \times 1} \frac{1}{2^5} = \frac{5}{16}$$

$$N.P(r=3) = \cancel{800}^{50} \times \frac{5}{\cancel{16}} = \boxed{250 \text{ families}}$$

(ii) $P(2B\ 3G) = P(r=2) = {}^5C_2 \left(\tfrac{1}{2}\right)^2 \left(\tfrac{1}{2}\right)^3 = {}^5C_2 \left(\tfrac{1}{2}\right)^5$

$$= \frac{5 \times \cancel{4}^2}{\cancel{2}} \frac{1}{2^5} = \frac{5}{16}$$

$$800 \times \frac{5}{16} = \boxed{250 \text{ families}}$$

(iii) $P(\text{No girl}) = P(r=5) = {}^5C_5 \left(\tfrac{1}{2}\right)^5 \left(\tfrac{1}{2}\right)^0$

$$= 1 \cdot \frac{1}{2^5} =$$

$$N.P(r=5) = 800 \times \frac{1}{2^5} = \frac{50}{2} = \boxed{25 \text{ families}}$$

(iv) $P(\text{At most 2 girl}) = P(r=5)+P(r=4)+P(r=3)$

OR $P(\text{at least 3 Boy})$

5 B	
4 B	1 G ✓
3 B	2 G ✓

$$= {}^5C_5\left(\tfrac{1}{2}\right)^5\left(\tfrac{1}{2}\right)^0 + {}^5C_4\left(\tfrac{1}{2}\right)^4\left(\tfrac{1}{2}\right)^1 + {}^5C_3\left(\tfrac{1}{2}\right)^3\left(\tfrac{1}{2}\right)^2$$

$$= \frac{1}{2^5}\left[{}^5C_5 + {}^5C_4 + {}^5C_3\right] = \frac{1}{2^5}\left[1+5+\frac{5\times \cancel{4}^2\times\cancel{3}}{\cancel{3}\times\cancel{2}}\right]$$

$$= \frac{16}{2^5} = \frac{1}{2}$$

$$800 \times P = \frac{1}{2}\times 800 = \boxed{400 \text{ families}}$$

Ex. Calculate Moment generating function of Discrete Binomial Distribution $P(x) = {}^nC_x p^x q^{n-x}$, $p+q=1$. Also find mean and Variance of Binomial Distribution.

$\mu = np$, $Var = npq$ } Bio Dist

$p+q=1$

Sol $M_x(t) = E\{e^{tx}\} = \sum e^{tx} p(x)$

$$= \sum_{x=0}^{n} e^{tx}\, {}^nC_x p^x q^{n-x} = \sum_{x=0}^{n} e^{tx}\, {}^nC_x p^x (1-p)^{n-x}$$

$(e^t p)^x$

$$= \sum_{x=0}^{n} {}^nC_x \underbrace{(e^t p)^x}_{b} \underbrace{(1-p)^{n-x}}_{a}$$

$$(a+b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \cdots + {}^nC_n a^0 b^n$$

$$M_x(t) = \left[pe^t + (1-p)\right]^n$$

$$= \sum_{r=0}^{n} {}^nC_r b^r a^{n-r}$$

$$\boxed{M_x(t) = [pe^t + q]}$$ $\sum_{r=0} {}^nC_r \ldots$

$$\mu = \text{Mean} = E(x) = \left.\frac{d}{dt} M_x(t)\right]_{t=0}$$

$$= \frac{d}{dt}\left[(pe^t + (1-p))^n \right]_{t=0} = n\left[[pe^t + (1-p)]^{n-1} pe^t \right]_{t=0}$$

$$= n[pe^0 + (1-p)]^{n-1} pe^0$$

$$\boxed{e^0 = 1}$$

$$= n[\cancel{p} + (1-\cancel{p})]^{n-1} p = np$$

$$\boxed{\mu = np}$$

$$\boxed{Var = E(x^2) - \{E(x)\}^2 = E(x^2) - \mu^2}$$

$$E(x^2) = \sum x^2 P(x)$$

$$E(x^2) = \left.\frac{d^2}{dt^2} M_x(t)\right]_{t=0}$$

$$= \frac{d}{dt}\left[n[pe^t + (1-p)]^{n-1} pe^t \right]_{t=0}$$

$$= np \frac{d}{dt}\left[\underbrace{[pe^t + (1-p)]^{n-1}}_{I} \underbrace{e^t}_{II} \right]_{t=0}$$

$$= np\left\{ (n-1)[pe^t + (1-p)]^{n-2} pe^t \cdot e^t + [pe^t + (1-p)]^{n-1} e^t \right]_{t=0}$$

$$= np\left\{ (n-1)[\cancel{p}.1 + 1 - \cancel{p}]^{n-2} p.1.1 + [\cancel{p}.1 + (1-\cancel{p})]^{n-1}. \right]$$

$$= np[(n-1)p + 1] = np[np - p + 1] = E(x^2)$$

$$Var = E(x^2) - \mu^2 = np[np - p + 1] - n^2p^2$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$

$$= np(1-p) = \boxed{npq}$$

Practice Question

Q-) Out of 800 families with 4 children each, how many families would be expected to have (i) 2 Boys, 2 Girl (ii) at least one Boy (iii) No girl (iv) At most two girl? Assume equal prob. for Boys and girls [2014]

[Ans :- (i) 300 , (ii) 750 (iii) 50

(iv) 550]

(2) If 10% of bolts produced by a machine are defective. Determine the probability that out of 10 bolts chosen at random (i) one (ii) None (iii) At most 2 bolts will be defective.

Unit-4 (Maths-4) (Lec-4) (By-Monika Mittal) (MM)

Statistical Techniques-II

Topic : Poission Distribution (Discrete R.V.)

In Binomial Distribution, we have already studied n and p.

If n is very large and p is very small. $n \to \infty$, $p \to 0$ $\boxed{np = \lambda}$

$n \to$ No of trials

$p \to$ prob of success

We get Poission Distribution. (limiting case of Binomial Dis.)

$$\to \boxed{P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!}}, \quad r = 0, 1, 2, 3 \ldots$$

(1) $P(x) \geq 0$

(2) $\sum P(x) = 1$

$$\sum_{r=0}^{\infty} P(r) = \sum_{r=0}^{\infty} \frac{e^{-\lambda}\lambda^r}{r!} = e^{-\lambda} + e^{-\lambda}\frac{\lambda^1}{1!} + e^{-\lambda}\frac{\lambda^2}{2!} + \ldots$$

$$= e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} \ldots\right]$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$= e^{-\lambda} e^{\lambda} = e^0 = 1$$

$\boxed{\lambda = np}$ $\lambda \to$ Poission Parameter

Mean $\to$ $\mu = E\{x\} = \sum_{r=0}^{\infty} x P(x) = \sum_{r=0}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!} = \boxed{\lambda}$

Variance $\to$ $Var = E\{x^2\} - [E(x)]^2 = E(x^2) - \mu^2$

$$= \boxed{\lambda}$$

Show that Poission Distribution is a particular limiting case of Binomial Distribution ,when p is very small and n is very large.

[2020-21, 2014,2013]

If we assume that as $n \to \infty$ and $p \to 0$ such that np always remains finite i.e $\boxed{np = \lambda}$ where λ is parameter

$n \to \infty$, $p \to 0$

$\boxed{np = \lambda}$

(10 Marks)

For Binomial Distribution

$$P(X=r) = {}^nC_r\, p^r q^{n-r}$$

$p + q = 1$

$q = 1 - p$

$np = \lambda$

$= \lambda/n$

$n \to \infty$

$$P(r) = \frac{n!}{r!\,(n-r)!}\, p^r (1-p)^{n-r}$$

$$= \frac{n(n-1)(n-2)\cdots(n-r+1)\cancel{(n-r)!}}{r!\,\cancel{(n-r)!}} \left(\frac{\lambda}{n}\right)^r \left(1-\frac{\lambda}{n}\right)^{n-r}$$

$$= \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!} \left(\frac{\lambda}{n}\right)^r \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^r}$$

$$= \frac{\lambda^r}{r!}\left[\frac{n(n-1)(n-2)\cdots(n-r+1)}{n^r} \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^r}\right]$$

$$= \frac{\lambda^r}{r!}\left[\left(\frac{n}{n}\right)\left(\frac{n-1}{n}\right)\left(\frac{n-2}{n}\right)\cdots\left(\frac{n-r+1}{n}\right) \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^r}\right]$$

$$= \frac{\lambda^r}{r!}\left[1\cdot\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{r-1}{n}\right) \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^r}\right]$$

As $n \to \infty$

$$P(r) = \frac{\lambda^r}{\lfloor r} \, 1 . \frac{e^{-\lambda}}{1}$$

$$\boxed{P(r) = \frac{e^{-\lambda} \lambda^r}{r!}}$$

$$\lim_{n\to\infty} \left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{\ }{n}\right)$$

$$= 1 . 1 \ldots 1$$

$$= 1$$

$1/\infty = 0$

$$\lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^r = 1$$

$$\lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^n = e^{-\lambda}$$

1^∞

$$\boxed{\lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n = e^x}$$

Recurrence Relation

$$P(r+1) = \frac{\lambda}{r+1} P(r) \quad , r = 0, 1, 2, 3 \ldots\ldots$$

$P(0) \rightarrow$ known

$P(1)$

By → Monika Mittal (MM)

Topic : Poission Distribution (Part 2)

Examples based upon Poission Distribution

Ex.1 If variance of Poission Distribution is 2. Find the probabilities for r = 1, 2, 3, 4 from recurrence Relation of Poission distribution, Also find $P(r \geq 4)$

[2013]

$$\boxed{P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!}}, \quad r = 0, 1, 2, 3 \ldots$$

$$\boxed{np = \lambda}$$

$n \to \infty$, $p \to 0$

Mean = Variance = $\boxed{\lambda = np}$

$$P(0) = \frac{e^{-2}\, 2^0}{0!}$$

$$\boxed{P(0) = e^{-2}} = .1353$$

Recurrence Relation $\boxed{P(r+1) = \frac{\lambda}{r+1} P(r)}$ — ?

Var = $\boxed{2 = \lambda}$ = np

$$P(r+1) = \frac{2}{r+1} P(r)$$

$r = 0$, $P(1) = \frac{2}{1} P(0) = 2 \times .1353 = .2706$

$r = 1$, $P(2) = \frac{\cancel{2}}{\cancel{2}} P(1) = .2706$

$r = 2$, $P(3) = \frac{2}{3} P(2) = \frac{2}{3} \times .2706 = .1804$

$r = 3$, $P(4) = \frac{\cancel{2}}{\cancel{4}_2} P(3) = \frac{1}{2} \times .1804 = .0902$

$P(r \geq 4) = 1 - [P(0) + P(1) + P(2) + P(3)]$

$= 1 - [.1353 + .2706 + .2706 + .1804]$

$= .1431$

Ex:→ In a certain factory manufacturing razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use suitable distribution to Calculate the approximate number of packets containing No defective, One defective and Two defective blades respectively in a consignment of 50,000 packets.

[AKTU 2018]

Sol $p(\text{defective}) = .002$

$n = 10$

$np = 10 \times .002 = \boxed{.02 = \lambda}$ ✓

No of Packets in Consignment $= 50,000$

→ $P(\text{No defective}) = P(r=0) = \frac{e^{-\lambda}\lambda^0}{0!} = \frac{e^{-.02}(.02)^0}{0!}$

$= .9802$ ✓

Approx. No of Packets $= 50,000 \times .9802$

$= \underline{49,010}$

→ $P(\text{one defective}) = P(r=1)$

$= \frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-.02}(.02)^1}{1!} = .01960$

Approx No of Packets $= 50,000 \times .01960$

$= 980$

→ $P(\text{Two defective}) = P(r=2)$

$= \frac{e^{-\lambda}\lambda^2}{2!} = \frac{e^{-.02}(.02)^2}{2!} = .00019604$

No of Packets $= 50,000 \times .00019604$

$= 9.802 \approx$ 10 Packets

Ex :- Fit a poission Distribution and Calculate theoritical frequencies

Deaths :	0	1	2	3	4
Frequencies :	122	60	15	2	1

[2014, 2010]

$P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!}$ $\quad r = 0,1,2,3,4$

Mean $= \lambda$ ✓

Var $= \lambda$

$\lambda = np$

x	f	fx
0	122	0
1	60	60
2	15	30
3	2	6
4	1	4
	200	100

$$\mu = \lambda = \frac{\Sigma fx}{\Sigma f} = \frac{100}{200} = \boxed{.5 = \lambda}$$

$$\boxed{N = 200}$$

r	$P(r) = \frac{e^{-\lambda}\lambda^r}{r!}$	$N.P(r)$	Theo fre
0	$P(0) = \frac{e^{-.5}(.5)^0}{0!}$	$200 \times \frac{e^{-.5}(.5)^0}{0!} = 121.306$	≈ 121
1	$P(1) = \frac{e^{-.5}(.5)^1}{1!}$	$200 \times \frac{e^{-.5}(.5)^1}{1!} = 60.65$	≈ 61
2	$P(2) = \frac{e^{-.5}(.5)^2}{2!}$	$200 \times \frac{e^{-.5}(.5)^2}{2!} = 15.163$	≈ 15
3	$P(3) = \frac{e^{-.5}(.5)^3}{3!}$	$200 \times \frac{e^{-.5}(.5)^3}{3!} = 2.527$	≈ 3
4	$P(4) = \frac{e^{-.5}(.5)^4}{4!}$	$200 \times \frac{e^{-.5}(.5)^4}{4!} = .3159$	≈ 0
			200

Ex:→ Find Moment generating function of Poission Distribution

$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$, $x = 0, 1, 2 \ldots$. Also find Mean and Variance of Distribution.

$$M_x(t) = E\{e^{tx}\} = \sum e^{tx} p(x)$$

$$= \sum_{x=0}^{\infty} e^{tx} \frac{e^{-\lambda}\lambda^x}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{(e^t\lambda)^x}{x!}$$

$\underbrace{e^{tx}\lambda^x}$

$(e^t\lambda)^x$

$$= e^{-\lambda}\left[1 + \frac{(e^t\lambda)}{1!} + \frac{(e^t\lambda)^2}{2!} + \frac{(e^t\lambda)^3}{3!} \cdots\right]$$

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots$$

$$= e^{-\lambda} \cdot e^{e^t\lambda} = e^{e^t\lambda - \lambda} = e^{\lambda(e^t-1)}$$

$$\boxed{M_x(t) = e^{\lambda(e^t-1)}}$$ ✓

$$\text{Mean} = \mu = E(x) = \frac{d}{dt} M_x(t)\Big]_{t=0}$$

$$= \frac{d}{dt}\left[e^{\lambda(e^t-1)}\right]_{t=0} = \left[e^{\lambda(e^t-1)} \frac{d}{dt}\lambda(e^t-1)\right]_{t=0}$$

$$= \left[e^{\lambda(e^t-1)}[\lambda e^t]\right]_{t=0} = e^{\lambda(e^0-1)}\lambda e^0 \quad \Big|\ e^0 = 1$$

$$= e^0 \cdot \lambda \cdot e^0 = \lambda$$

$$\boxed{\mu = \lambda}$$

$$\text{Variance} = E(x^2) - [E(x)]^2 = E(x^2) - \mu^2$$

$$E(x^2) = \frac{d^2}{dt^2} M_x(t)\Big]_{t=0} = \frac{d}{dt}\Big[\lambda \underbrace{e^t}_{\text{I}} \cdot \underbrace{e^{\lambda(e^t-1)}}_{\text{II}}\Big]_{t=0}$$

$$= \lambda\left[e^t \cdot e^{\lambda(e^t-1)} + e^t \cdot e^{\lambda(e^t-1)} \lambda e^t\right]_{t=0}$$

$$= \lambda\left[e^0 e^{\lambda(e^0-1)} + e^0 e^{\lambda(e^0-1)} \lambda e^0\right]$$

$$= \lambda\left[1 \cdot e^0 + e^0 \cdot \lambda\right] = \lambda(\lambda+1)$$

$$\text{Var} = E(x^2) - \mu^2 = \lambda(\lambda+1) - \lambda^2$$

$$= \cancel{\lambda^2} + \lambda - \cancel{\lambda^2}$$

$$= \lambda$$

$$\boxed{\text{Var} = \lambda}$$

Unit-4 (Maths-4) (Lec-6) (By- Monika Mittal (MM))

Statistical Techniques-II

Topic : Normal Distribution

The Normal Distribution is continuous distribution.It is limiting case of Binomial Distribution when no of trials n is very large and prob. Of success is close to 1/2 .

$p \approx 1/2$

$X \sim N(\mu, \sigma^2)$ $\mu \to$ Mean, σ^2 = Var

$$\boxed{f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}, \quad -\infty < x < \infty \to \text{Prob Den. fn (PDF)}$$

Here μ, σ are two parameters of Normal distribution

μ - Mean

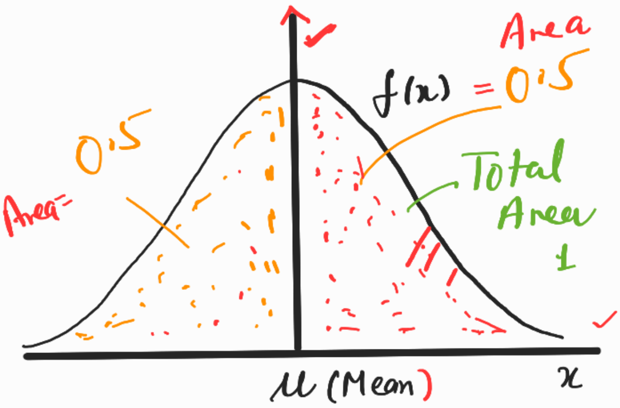
σ - Standard Deviation.

The graph of Normal Distribution is called Normal curve.

It is bell-shaped and symmetrical about Mean μ

In Normal Distribution, mean μ, mode and median coincide.

$\Rightarrow$ Total Area Under the curve above x-axis is 1



Normal curve

Basic Properties

PDF $f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

(i) $f(x) \geq 0$ (ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$ = Area

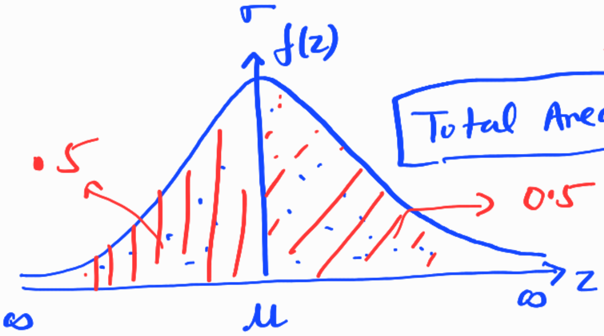
* The Normal Distribution is symmetrical about its mean.
* The mean, mode, median of Normal distribution coincide.
* Total area under curve is 1.

Standard Normal Distribution

$Z \sim N(\mu = 0, \sigma^2 = 1)$

$$\boxed{f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} z^2}}$$

$Z = \frac{x-\mu}{\sigma}$

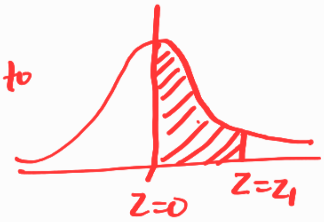


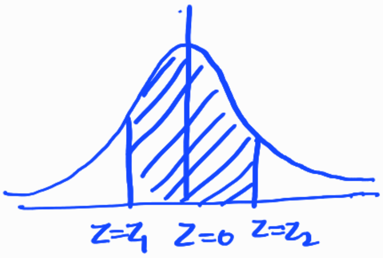
Prob = Area Under the Curve

$P(z_1 \leq Z \leq z_2) = \int_{z_1}^{z_2} f(z)\,dz$ = Area Under the curve b/w $z = z_1$ and $z = z_2$

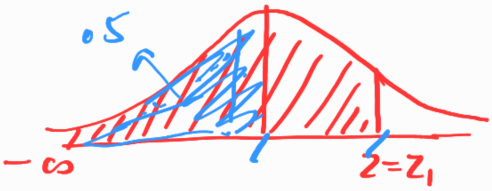
$\boxed{\text{By Using Table}}$

$P(0 < Z < z_1)$ = Area $Z=0$ to $Z=z_1$



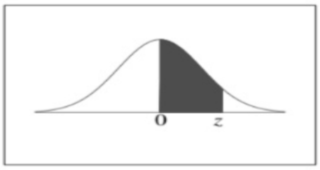


$P(-\infty < Z < z_1)$



$= .5 + P(0 < Z < z_1)$

Standard Normal Distribution Table



Area Under the Normal curve $= \frac{1}{\sqrt{2\pi}} \int_0^z e^{-z^2/2}\, dz$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830

.6

$Z=$

$.67$

$.07$

1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Ex. If X is Normal Variate with mean 80 and standard deviation 10 find Probability

$P(X \leq 100)$

$\mu = 80, \quad \sigma = 10$

$$Z = \frac{x-\mu}{\sigma}$$

$P(X \leq 100)$

$$x = 100, \quad Z = \frac{100-80}{10} = \frac{20}{10} = 2$$

$x = 100, \quad Z = 2$

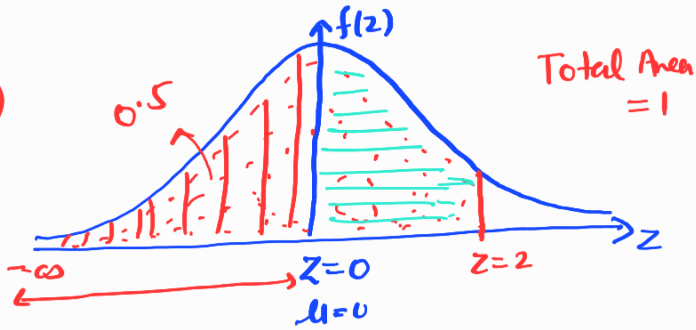
$P(X \leq 100) = P(Z \leq 2)$

$=$ Area b/w $Z = -\infty$ to $Z = 2$

$= P(-\infty < Z < 0) + P(0 < Z < 2)$

$= .5 + .4772$

$= \underline{\underline{.9772}}$



Ex. A sample of 100 dry battery cells tested to find the length of life produced the following result:

$\bar{x} = 12$ hours, $S.D = \sigma = 3$ hours

Assuming the data to be normally distributed, what % of battery cells are expected to have life

(i) More than 15 hours

[2018, 20-21]

(i) More than

(ii) less than 6 hours

(iii) between 10 and 14 hours

Sol $\mu = 12$, $\sigma = 3$ Normally Distributed

let x be length of life of battery cell

(i) $P(x > 15) = P(z > 1)$

$$z = \frac{x-\mu}{\sigma}$$

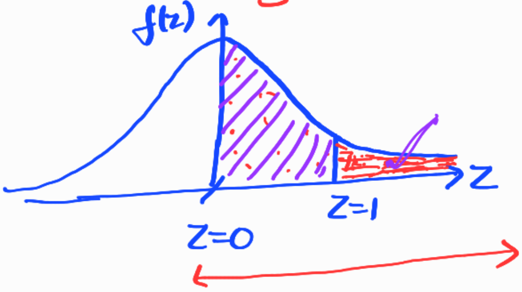
$$z = \frac{15-12}{3} = \frac{3}{3} = 1$$

$= P(0 < z < \infty) - P(0 < z < 1)$

$= .5 - .3413$

$= .1587$

$= 15.87\%$



(ii) $P(x < 6)$

$$z = \frac{x-\mu}{\sigma} = \frac{6-12}{3} = -\frac{6}{3} = -2$$

$= P(z < -2)$

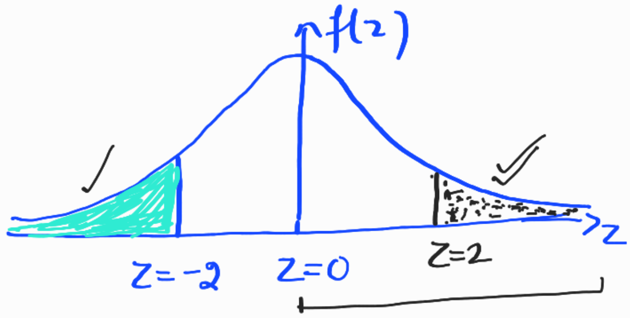
$= P(z > 2)$

$= .5 - P(0 < z < 2)$

$= .5 - .4772$

$.0228$

$= 2.28\%$



(iii) $P(10 < x < 14)$

$$z_1 = \frac{x-\mu}{\sigma} = \frac{10-12}{3} = -2/3$$

$= -.67$

$$z_2 = \frac{x-\mu}{\sigma} = \frac{14-12}{3} = 2/3$$

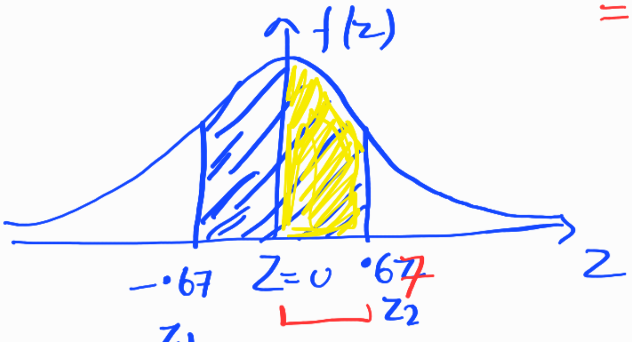
$= .67$

$= P(-.67 < z < .67)$

$= 2\,P(0 < z < .67)$

$= 2 \times .2486$

$= .4972$



$= 49.72\%$

Ex. In a Normal Distribution, 31% of the items are under 45 and 8% are over 64. Find mean and standard deviation of distribution. It is given that $f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-x^2/2}\, dx$, then $f(0.5) = 0.19$

and $\boxed{f(1.4) = 0.42}$

[2019, 2018]

$Z = \frac{x-\mu}{}$

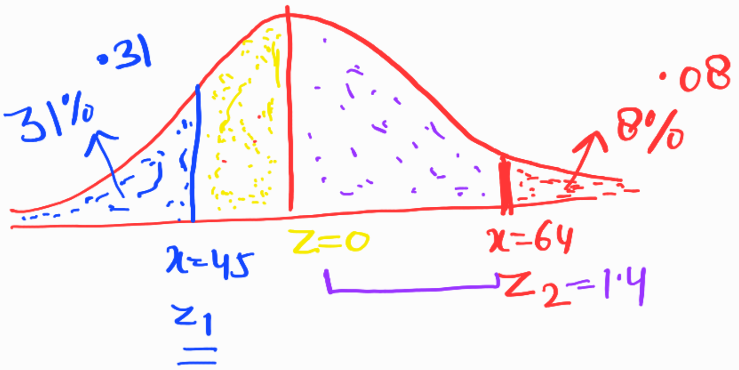
$f(x) = \frac{1}{\phantom{\sigma\sqrt{2\pi}}} e^{-\left(\frac{x-\mu}{\sigma}\right)^2}$

31% under 45 = Area

$$Z = \frac{x-\mu}{\sigma}, \quad \mu = 0, \ \sigma = 1$$

$x = 45$

$$Z_1 = \frac{45-\mu}{\sigma}$$



$$P(-\infty < Z < Z_1) = .31$$

$$P(-Z_1 < Z < 0) = .5 - .31 = .19$$

$$[f(0.5) = .19]$$

$$Z_1 = -.5$$

$$-.5 = \frac{45-\mu}{\sigma} \Rightarrow \mu - .5\sigma = 45 \quad \text{--- (1)}$$

$$P(0 < Z < Z_2) = .5 - .08 = .42$$

$$f(1.4) = .42$$

$$Z_2 = \frac{x-\mu}{\sigma}$$

$$1.4 = \frac{64-\mu}{\sigma} \Rightarrow \mu + 1.4\sigma = 64 \quad \text{--- (2)}$$

$$\cancel{\mu} - .5\sigma = 45$$
$$\underline{-\cancel{\mu} \mp 1.4\sigma = -64}$$
$$\cancel{-}1.9\sigma = \cancel{-}19$$

$$\boxed{\sigma = 10} \qquad \boxed{\mu = 50}$$

Unit-4 (Maths-4) (Lec-7) By- Monika Mittal (MM)

Statistical Techniques-II (PART-1)

Topic : Conditional Probability,Baye's Theorem

Probability

$$P(E) = \frac{\text{No of favourable cases}}{\text{Total No of Cases}}$$

→ Probability of happening event E

Addition Theorem of Probablities

If A and B are two events then Probability of happening A or B

$$\boxed{\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \end{aligned}}$$

If A and B are Mutually Exclusive events then $A \cap B = \phi$ so

$$\boxed{P(A \cup B) = P(A) + P(B)}$$

$$\boxed{\begin{aligned} P(A \cup B \cup C) = {} & P(A) + P(B) + P(C) - P(A \cap B) - \\ & P(B \cap C) - P(C \cap A) + P(A \cap B \cap C) \end{aligned}}$$

Conditional Probability

The prob. of happening of an Event A ,when event B is already happened is called conditional probability.

It is denoted by

$$P(A|B)$$

Multiplication Law of Probability

The prob. of occurrence of two events simultaneously

$$P(A \cap B) = P(A) \cdot P(B/A)$$

$P(B/A) \rightarrow$ Conditional Probability of occurrence of B when A already happened.

If A and B are Mutually exclusive

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(AB) = P(A) \cdot P(B)$$

If $A_1, A_2, A_3 \ldots A_n$ are independent Events then

$$P(A_1 . A_2 \ldots A_n) = P(A_1).P(A_2).P(A_3) \ldots P(A_n)$$

Ex. $\rightarrow$ A can hit a target 4 times in 5 shots, B 3 times in 4 shots, C 2 times in 3 shots. What is the prob. that at least two shots hits. ?

$P(A \text{ hit target}) = \frac{4}{5}$, $P(B \text{ hit target}) = \frac{3}{4}$

$P(C \text{ hit target}) = 2/3$

$E_1 \Rightarrow$ (i) A, B, C hit target

$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C) = \cancel{\frac{4}{5}} \cdot \cancel{\frac{3}{4}} \cdot \frac{2}{\cancel{3}} = \frac{2}{5}$ ✓

$E_2 \Rightarrow$ (ii) Prob (A, B hit, C miss target) $= \frac{4}{5} \cdot \frac{3}{4} \cdot (1 - 2/3)$

$= \cancel{\frac{4}{5}} \cdot \cancel{\frac{3}{4}} \cdot \frac{1}{\cancel{3}} = \frac{1}{5}$ ✓

$E_3 \Rightarrow$ (iii) Prob (B, C hit, A miss target)

$= \frac{3}{4} \cdot \frac{2}{3} \cdot \left(1 - \frac{4}{5}\right) = \frac{\cancel{3}}{\cancel{4}_2} \cdot \frac{\cancel{2}}{\cancel{3}} \cdot \frac{1}{5} = \frac{1}{10}$ ✓

$E_4 \Rightarrow$ (iv) Prob(A, C hit, B miss target)

$= \frac{4}{5} \cdot 2/3 \left(1 - \frac{3}{4}\right) = \frac{\cancel{4}}{5} \cdot \frac{2}{3} \cdot \frac{1}{\cancel{4}} = \frac{2}{15}$ ✓

Req Prob $= P(E_1 \cup E_2 \cup E_3 \cup E_4) = P(E_1) + P(E_2) + P(E_3) + P(E_4)$

$= \frac{2}{5} + \frac{1}{5} + \frac{1}{10} + \frac{2}{15}$

$= \frac{25}{30} = 5/6$

Ex. → An Urn contains 10 White and 3 Black balls while another urn contains 3 white and 5 Black balls. Two balls are drawn from first urn and put it into second urn and then a ball is drawn from the latter. What is the probability that it is white ball?

II Urn: 3 W, 5 Blk

I Urn: 10 W, 3 Blk

Put into II

1 B drawn

$P(W) = ?$

(2B) drawn

$E_1 \Rightarrow 1W, 1\ Blk \Rightarrow P(1W\ 1Black) = \dfrac{{}^{10}C_1 \times {}^{3}C_1}{{}^{13}C_2} = \dfrac{10 \times \cancel{3} \times \cancel{2}}{13 \times \cancel{12}_{\cancel{4}\,2}} = 10/26 = 5/13$

$E_2 \Rightarrow 2W \quad P(2W) = \dfrac{{}^{10}C_2}{{}^{13}C_2} = \dfrac{10 \times 9 \quad \cancel{2!}}{\cancel{2!}\ 13 \times 12} = \dfrac{15}{26}$

$E_3 \Rightarrow 2\ Blk \quad P(2Blk) = \dfrac{{}^{3}C_2}{{}^{13}C_2} = \dfrac{\cancel{3} \times \cancel{2} \quad \cancel{2!}}{\cancel{2!}\ 13 \times \cancel{12}\ \cancel{6}\ 2} = \dfrac{1}{26}$ ✓

II Urn

$E_1 \to (1)$	4W	6 Blk	$\to$	$P(W/E_1) = \dfrac{{}^{4}C_1}{{}^{10}C_1} = \dfrac{4}{10}$ ✓
$E_2 \to (2)$	5W	5 Blk.	$\to$	$P(W\mid E_2) = \dfrac{{}^{5}C_1}{{}^{10}C_1} = \dfrac{5}{10}$ ✓
$E_3 \to (3)$	3W	7 Blk.	$\to$	$P(W\mid E_3) = \dfrac{{}^{3}C_1}{{}^{10}C_1} = \dfrac{3}{10}$ ✓

OR

Req Prob $\to$ $P(E_1).P(W|E_1) + P(E_2).P(W|E_2) + P(E_3).P(W|E_3)$

$$= \frac{5}{13}\cdot\frac{4}{10} + \frac{15}{26}\,\frac{5}{10} + \frac{1}{26}\,\frac{3}{10}$$

$$= \frac{40 + 75 + 3}{26 \times 10} = \frac{118}{260}$$

$$= 59/130$$

BAYE'S THEOREM

If $E_1, E_2, \ldots E_n$ are mutually exclusive and exhaustive events with $P(E_i)$, $i = 1, 2 \ldots n$ of a random experiment

$$P(E_i/A) = \frac{P(E_i) \cdot P(A|E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A|E_i)} = \frac{P(E_i) P(A|E_i)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + \cdots + P(E_n) P(A|E_n)}$$

$P(E_i|A)$ = Prob of occurrence of E_i when event A has already occurred.

$$P(E_i|A) = \frac{P(E_i \cap A)}{P(A)}$$

Ex. Two Urns contain 4 white, 6 Blue, and 4 white 5 Blue balls respectively. One of the urn is selected at random and a ball is drawn from it. If the ball drawn is white, find Prob. that it is drawn from

(i) first Urn (ii) Second Urn.

I^{st} Urn: 4W, 6B

II^{nd}: 4W, 5B

$E_1 = I^{st}$ chosen $\qquad E_2 = II^{nd}$ chosen $\qquad A :\Rightarrow$ Ball is white

$P(E_1) = 1/2 \qquad P(E_2) = 1/2$

$$P(A|E_1) = \frac{{}^4C_1}{{}^{10}C_1} = \frac{4}{10} = \frac{2}{5}$$

$$P(A|E_2) = \frac{{}^4C_1}{{}^9C_1} = \frac{4}{9}$$

(i) $$P(E_1|A) = \frac{P(E_1 \cap A)}{P(A)} = \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + \ldots}$$

$$P(A) \qquad P(E_2)\,P(A|E_2)$$

$$= \frac{\frac{1}{2}\cdot\frac{2}{5}}{\frac{1}{2}\cdot\frac{2}{5} + \frac{1}{2}\cdot\frac{4}{9}} = \frac{\frac{2}{10}}{\frac{1}{5}+\frac{2}{9}} = \frac{2/10}{\frac{19}{45}} = \frac{2\times 45}{19\times 10}$$

$$= \frac{9}{19}$$

$$(ii)\quad P(E_2|A) = \frac{P(E_2\cap A)}{P(A)} = \frac{P(E_2).P(A|E_2)}{P(E_1).P(A|E_1)+P(E_2).P(A|E_2)}$$

$$= \frac{\left(\frac{1}{2}\right)\cdot\frac{4}{9}}{\frac{19}{45}} = \frac{\frac{2}{9}}{19/45}$$

$$\frac{2\times 45}{9\times 19} = \frac{10}{19}$$

Ex. In a bolt factory, machines A, B, C manufacture respectively 25%, 35% and 40% of the total. Of their output 5, 4 and 2 % are defective bolts. A bolt is drawn at random from product and is found to be defective. What is the prob. that it was manufactured by machine B?

Bolt Factory

A	B	C
25%	35%	40%
def. 5%	def. 4%	def 2%

E_1 : Bolt selected at random is manufactured by machine A ✓

E_2 : Bolt selected at random is manufactured by Machine B ✓

E_3 : Bolt selected at random is manufactured by Machine C ✓

D : event of being defective ✓

$P(E_1) = \cdot 25$ $\quad$ $P(E_2) = \cdot 35$ $\quad$ $P(E_3) = \cdot 40$

$$P(D \mid E_1) = \cdot 05$$
$$P(D \mid E_2) = \cdot 04$$
$$P(D \mid E_3) = \cdot 02$$

$$P(E_2 \mid D) = \frac{P(E_2 \cap D)}{P(D)} = \frac{P(E_2) \cdot P(D \mid E_2)}{P(E_1) \cdot P(D \mid E_1) + P(E_2) \cdot P(D \mid E_2) + P(E_3) \cdot P(D \mid E_3)}$$

$$= \frac{(\cdot 35)(\cdot 04)}{(\cdot 25)(\cdot 05) + (\cdot 35)(\cdot 04) + (\cdot 40)(\cdot 02)}$$

$$= \underline{\underline{\cdot 41}}$$